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Engineering Fluid Mechanics Solution Manual

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Contents

Book Description:

Title – Engineering Fluid Mechanics Solution Manual

Author – Prof. T.T. Al-Shemmeri

Fluid Mechanics is an essential subject in the study of the behaviour of fluids at rest and when in motion.

The book is complimentary follow up for the book "Engineering Fluid Mechanics" also published on BOOKBOON, presenting the solutions to tutorial problems, to help students the option to see if they got the correct answers, and if not, where they went wrong, and change it to get the correct answers.

Author Details:

Prof Tarik Al-Shemmeri – BSc, MSc, PhD, CEng

Professor of Renewable Energy Technology at Staffordshire University.
Current research interests in Renewable Energy and Environmental Technology.
Lecturing topics include: Energy management and Power generation.

1 Chapter One Tutorial Problems

1.1 Show that the kinematic viscosity has the primary dimensions of L^2T^{-1}.

Solution:

The kinematic viscosity is defined as the ratio of the dynamic viscosity by the density of the fluid.

The density has units of mass (kg) divided by volume (m^3); whereas the dynamic viscosity has the units of mass (kg) per meter (m) per time (s).

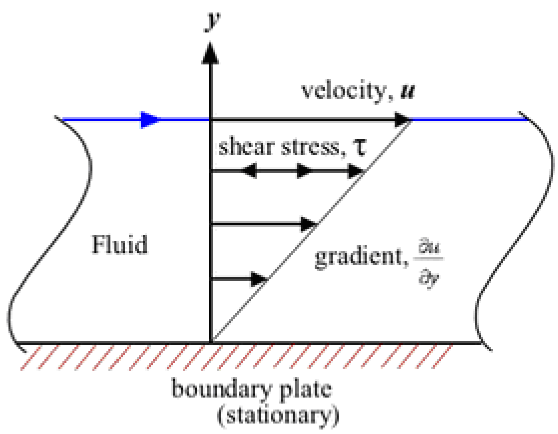
Hence:

$$\upsilon = \frac{\mu}{\rho} = \frac{ML^{-1}T^{-1}}{ML^{-3}} = L^2T^{-1}$$

1.2 In a fluid the velocity measured at a distance of 75mm from the boundary is 1.125m/s. The fluid has absolute viscosity 0.048 Pa s and relative density 0.913. What is the velocity gradient and shear stress at the boundary assuming a linear velocity distribution? Determine its kinematic viscosity.

[Ans: 15 s^{-1}, 0.72Pa.s; 5.257×10^{-5} m^2/s]

Solution:



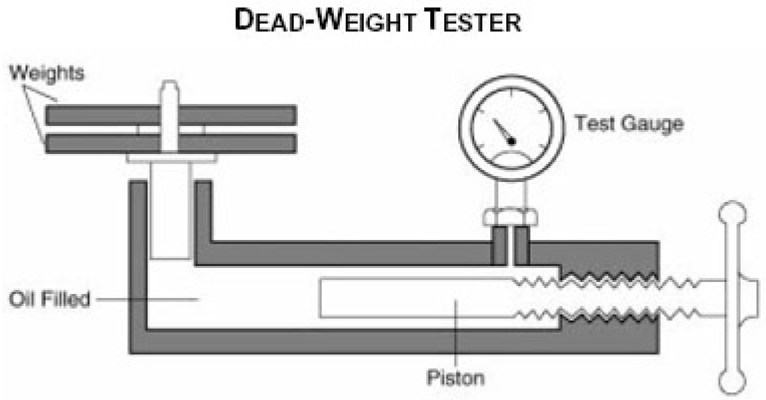
$$Gradient = \frac{dV}{dy} = \frac{1.125}{0.075} = 15\ s^{-1}$$

$$\tau = \mu \frac{dV}{dy} = 0.048x15 = 0.720\ Pa.s$$

$$\upsilon = \frac{\mu}{\rho} = \frac{0.048}{913} = 5.257x10^{-5}\ m^2 / s$$

1.3 A dead-weight tester is used to calibrate a pressure transducer by the use of known weights placed on a piston hence pressurizing the hydraulic oil contained. If the diameter of the piston is 10 mm, determine the required weight to create a pressure of 2 bars.

[Ans: 1.6 kg]



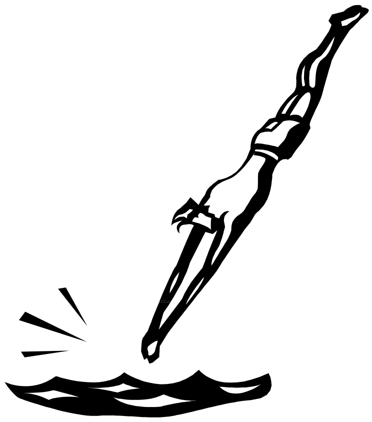
Solution:

(a) P = F/A

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} x0.010^2 = 7.854x10^{-5}\ m^2$$

Hence the weight = P x A / g
= 2 x 10^5 x7.854x10^{-5} / 9.81
= 1.60 kg

1.4 How deep can a diver descend in ocean water without damaging his watch, which will withstand an absolute pressure of 5.5 bar?
Take the density of ocean water, ρ = 1025 kg/m^3.



[Ans: 44.75 m]

Solution:

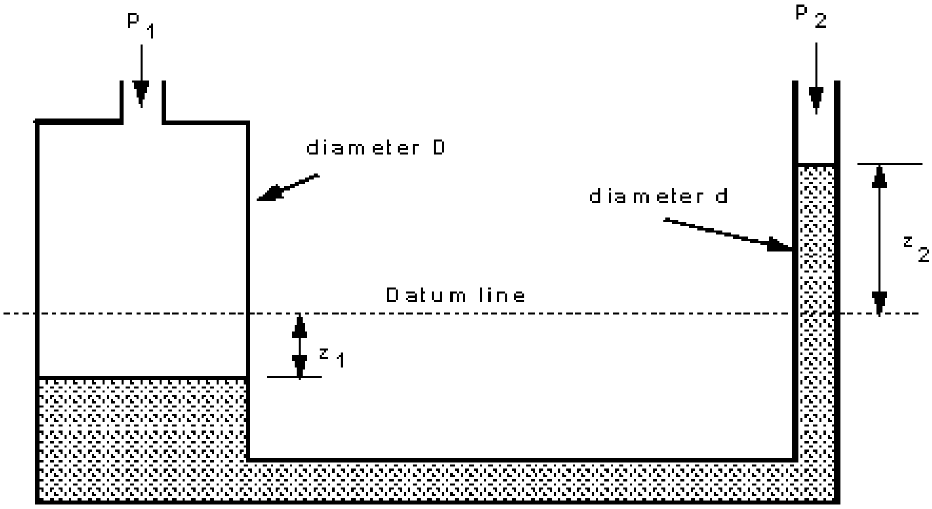
Use the static equation: $p = \rho g h$

Hence the depth can be calculated as:

$$h = \frac{P}{\rho.g} = \frac{(5.5-1)x10^5}{1025x9.81} = 44.75m\,Water$$

1.5 The U-tube manometer shown below, prove that the difference in pressure is given by:

$$P_1 - P_2 = \rho.g.z_2\left[1+\left(\frac{d}{D}\right)^2\right]$$



Solution:

The relationship between liquid columns and the area of cross section is based on the conservation of matter, ie continuity equation, hence:

volume 1 = *volume* 2

$$z_1 x A_1 = z_2 x A_2$$

$$hence \rightarrow z_1 = \frac{z_2 x(\pi/4) x d^2}{(\pi/4) x D^2} = z_2 x(\frac{d}{D})^2$$

For the "U" tube manometer that the height different in the two columns gives the pressure difference, therefore:

$$F1 = F2$$
$$P_1 + \rho.g.z_1 = P_2 + \rho.g.z_2$$

hence

$$P_1 - P_2 = \rho.g.(z_2 - z_1)$$

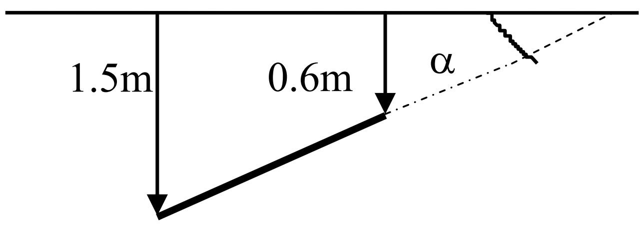
$$= \rho.g.z_2 x[1 + (d/D)^2]$$

Clearly if D is very much larger than *d* then $(d/D)^2$ is very small so

$$P_1 - P_2 = \rho.g.z_2$$

1.6 A flat circular plate, 1.25 m diameter is immersed in sewage water (density 1200 kg/m^3) such that its greatest and least depths are 1.50 m and 0.60 m respectively. Determine the force exerted on one face by the water pressure,

[Ans: 15180 N]



Solution:

Area of laminar

$$A = \frac{1}{4}\pi(1.25)^2 = 1.228 \text{ m}^2$$

Depth to centroid

$$h_c = \frac{1}{2}(0.60 + 1.50) = 1.05 \text{ m}$$

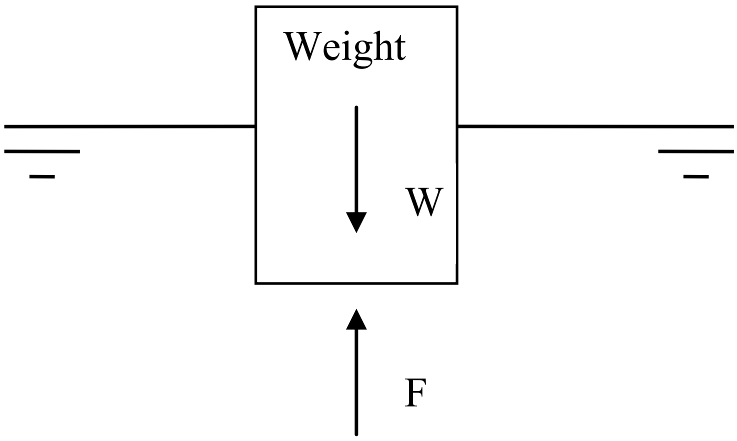
Resultant Force

$$F = \rho \, g \, A \, h_c = 9.81 \text{ x } 1200 \text{ x } 1.228 \text{ x } 1.05$$
$$= 15180 \text{ N}$$

1.7 A rectangular block of wood, floats with one face horizontal in a fluid (RD = 0.9). The wood's density is 750 kg/m^3. Determine the percentage of the wood, which is not submerged.

[Ans: 17%]

Solution:



For stable condition

Upthrust = weight force or

F = W

The Upthrust due to Buoyancy = $\rho_{seawater}\, g\, V_x$

The total weight of submersed wood = $\rho_{wood}\, g\, V_L$

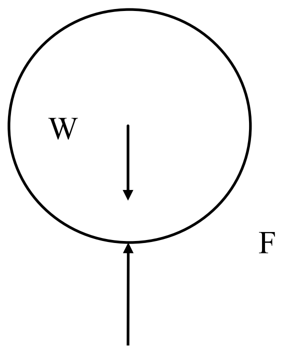
Therefore the portion of block that is NOT submerged is

$1 - V_x/V_L = (\rho_{wood} / \rho_{water}) = 1 - 750 / 900 = 17\%$

1.8 An empty balloon and its equipment weight 50 kg, is inflated to a diameter of 6m, with a gas of density 0.6 kg/m^3. What is the maximum weight of cargo that can be lifted on this balloon, if air density is assumed constant at 1.2 kg/m^3?

[Ans: 17.86 kg]

Solution:



Since the system is stable according to Newton's second law of motion:

Upthrust = Weight force + Payload

The Upthrust is F $= \rho \text{ x } g \text{ x} V_L = 1.2 \text{ x } 9.81 \text{ x}(\pi\text{x}(4/3)\text{x}3^3) = 1331.4$ N

Weight of balloon gas $= \rho \text{ x} V_L = 0.6 \text{ x } 9.81 \text{ x}(\pi\text{x}(4/3)\text{x}3^3) = 666$ N

Weight of equipment = 50 kg = 50x9.81 = 490.5 N

Therefore the payload that can be lifted

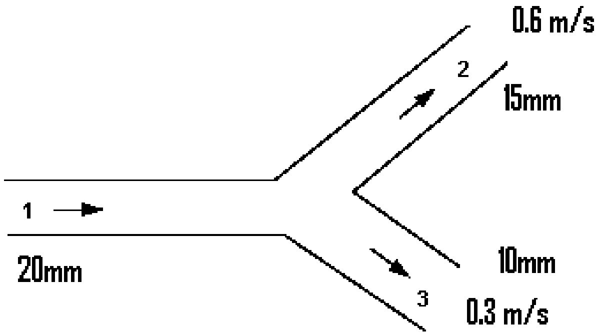
P = F - W
= 1331.4 – (666 + 490.5)
= 175 N = 17.9 kg.

2 Chapter Two Tutorial Problems

2.1 A 20 mm dam pipe forks, one branch being 10 mm in diameter and the other 15 mm in diameter. If the velocity in the 10 mm pipe is 0.3 m/s and that in the 15 mm pipe is 0.6 m/s, calculate the rate of flow in cm^3/s and velocity in m/s in the 20 mm diameter pipe.

(129.6 cm^3/s, 0.413 m/s)

Solution:



Total mass flow into the junction = Total mass flow out of the junction

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3$$

When the flow is incompressible (e.g. if it is water) $\rho_1 = \rho_2 = \rho$

$$\begin{aligned} Q &= A_2 V_2 + A_3 V_3 \\ &= (\pi/4) D_2^2 x\, V_2 + (\pi/4) D_3^2 x\, V_3 \\ &= (\pi/4)\, 0.01^2 x\, 0.3 + (\pi/4)\, 0.015^2 x\, 0.6 \\ &= 2.356x10^{-5} + 10.6x10^{-5} \\ &= 12.96\, x10^{-5}\ m^3/s \end{aligned}$$

since

$$Q = A_1\ x\, V_1$$

$$\therefore V_1 = \frac{Q}{\frac{\pi}{4} x 0.02^2} = 0.4125\ m/s$$

2.2 Water at 36 m above sea level has a velocity of 18 m/s and a pressure of 350 kN/m². Determine the potential, kinetic and pressure energy of the water in metres of head. Also determine the total head.

Ans (35.68 m, 16.5 m, 36 m, 88.2 m)

Solution:

Take each term separately

$$\frac{p_1}{\rho g} = \frac{350x10^3}{1000x9.81} = 35.678m$$

$$\frac{V_1^2}{2g} = \frac{18^2}{2x9.81} = 16.514m$$

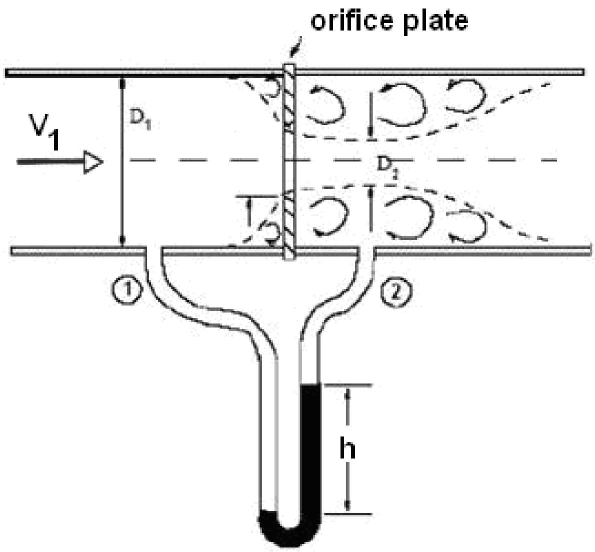
$$z_1 = 36m$$

The total head = 35.678 + 16.514 + 36 = 88.192 m

2.3 The air supply to an engine on a test bed passes down a 180 mm diameter pipe fitted with an orifice plate 90 mm diameter. The pressure drop across the orifice is 80 mm of paraffin. The coefficient of discharge of the orifice is 0.62 and the densities of air and paraffin are 1.2 kg/m^3 and 830 kg/m^3 respectively. Calculate the mass flow rate of air to the engine.

Ans (0.16 kg/s)

Solution:



(i) $p_1 - p_2 = \rho_m \text{ x g x h}$

$= 830 \text{ x } 9.81 \text{ x } 0.080 = 651 \text{ Pa}$

(ii) $$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1-(A_2/A_1)^2]}} = \sqrt{\frac{2x651.4}{1.2x[1-(90/180)^4]}} = 34.03\ m/s$$

(iii) $m = C_d V_2 A_2 \text{ x } \rho$

$$A_2 = \frac{\pi}{4} x0.09^2 = 6.36x10^{-3}\ m^2$$

Q $= 0.62 \text{ x } 34.03 \text{ x } 0.00636 \text{ x } 1.2$

$= 0.161$ kg/s

2.4 Determine the pressure loss in a 100 m long, 10 mm diameter smooth pipe if the flow velocity is 1 m/s for:

a) air whose density 1.0 kg/m^3 and dynamic viscosity 1 x 10^{-5} Ns/m^2.

b) water whose density 1000^3 kg/m^3 and dynamic viscosity 1 x 10^{-3} Ns/m^2.

Ans: (320 N/m^2, 158 kN/m^2)

Solution:

a) For air

Re = ρ V D/μ = 1 x 1 x 0.01/ 1 x 10^{-5} = 1000 i.e. laminar

f = 16/Re=16/(1000)= 0.016

therefore $$h_f = 4xfx\frac{L}{D}\frac{v^2}{2g}$$

$$h_f = 4x0.016x\frac{100}{0.01}x\frac{1^2}{19.62} = 32.6\ m$$

$$P = \rho.g.h_f = 1.0x9.81x32.6 = 320\ N/m^2$$

b) for water

Re = ρ V D/μ

= 1000 x 1 x 0.01/ 1 x 10^{-3} = 10^4 i.e. turbulent flow

f = 0.079/$Re^{0.25}$

= 0.079/$(10^4)^{0.25}$ = 0.0079

therefore $$h_f = 4xfx\frac{L}{D}\frac{v^2}{2g}$$

$$hf = 4x0.0079x\frac{100}{0.01}x\frac{1^2}{19.62} = 16.1\ m$$

$$P = \rho.g.h_f = 1000x9.81x16.1 = 157941 = 158\ kN/m^2$$

2.5 Determine the input power to an electric motor (η_m = 90%) supplying a pump (η_p = 90%) delivering 50 l/s of water (ρ = 1000 kg/m^3, μ = 0.001 kg/ms) between two tanks with a difference in elevation of 50m if the pipeline length is 100m long in total of 150 mm diameter, assume a friction factor of 0.008 and neglect minor losses.

Ans: (33.6 kW)

Solution:

$V = Q/A = 0.05/(\pi \text{ x } 0.15^2/4) = 2.83$ m/s

$Re = \rho VD/\mu = 1000 \text{ x } 2.83 \text{ x } 0.15/0.001 = 4.244 \text{ x } 10^5$

k/D = 0.15/150 = 0.001; From Moody diagram f = 0.0051

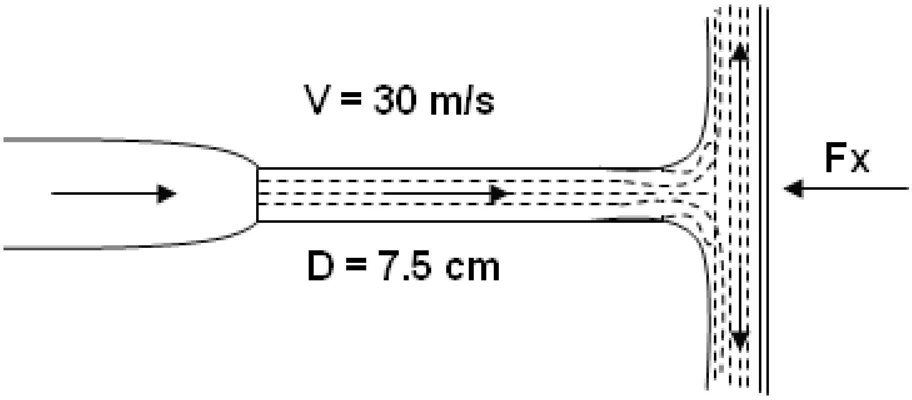
$$h_f = \frac{4\,f\,L}{D}\frac{v^2}{2g} = \frac{4x0.008x100}{0.15}\,x\,\frac{2.83^2}{2x9.81} = 8.708\ m$$

$h_{sys} = H_f + H_z + H_o = 8.708 + 50 + 0 = 58.808$ m

$$\text{Input power } P = \frac{\rho.g.Q.H_{sys}}{\eta_m\eta_p} = \frac{1000x9.81x0.05x58.708}{0.9x0.9} = 33.551\,kW$$

2.6 A jet of water strikes a stationary flat plate "perpendicularly", if the jet diameter is 7.5 cm and its velocity upon impact is 30 m/s, determine the magnitude and direction of the resultant force on the plate, neglect frictional effect and take water density as 1000 kg.m^3.

Ans (3970 N)



Solution:

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}x0.075^2 = 4.418x10^{-3}\,m^2$$

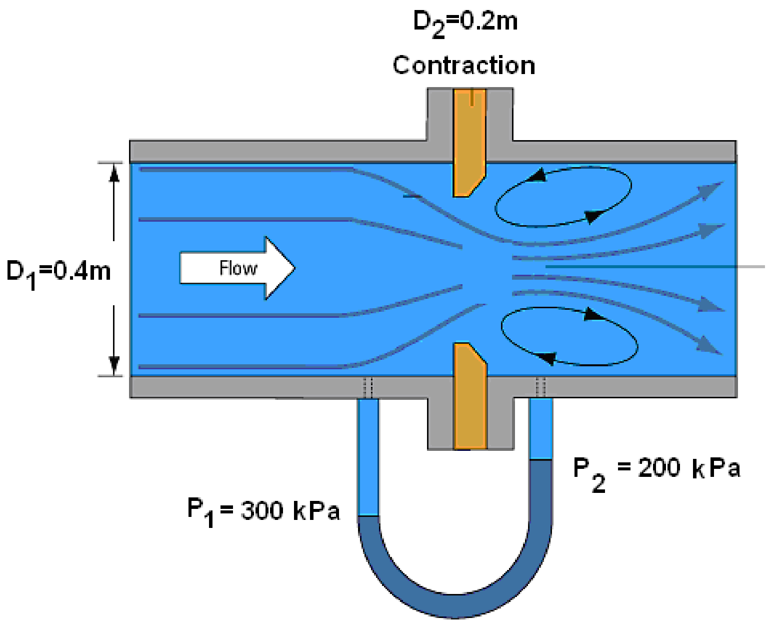
$$Fx = \rho A V^2(1 - \cos\theta)$$

$$Fx = 1000x4.418x10^{-3}x30^2x(1-0)$$

$$Fx = 3976\ N$$

2.7 A horizontally laid pipe carrying water has a sudden contraction in diameter from 0.4m to 0.2m respectively. The pressure across the reducer reads 300 kPa and 200 kPa respectively when the flow rate is 0.5 m³/s. Determine the force exerted on the section due to the flow, assuming that friction losses are negligible.

Ans: (25.5 kN)



Solution:

$$A_1 = \frac{\pi}{4} x0.4^2 = 0.1256\, m^2$$

$$A_2 = \frac{\pi}{4} x0.2^2 = 0.0314\, m^2$$

$$v_1 = \frac{Q}{A_1} = \frac{0.5}{\frac{\pi}{4} x0.4^2} = 3.979\, m/s$$

$$v_2 = \frac{Q}{A_2} = \frac{0.5}{\frac{\pi}{4} x0.2^2} = 15.915\, m/s$$

$$-F_x + p_1 A_1 - p_2 A_2 \cos\theta = \rho \dot{V} (V_2 \cos\theta - V_1)$$

$$-Fx + 300x10^3 x0.1256 - 200x10^3 x0.0314x1 = 1000x0.5x(15.915-3.979)$$

$$-Fx + 37680 - 6283 = 5968$$

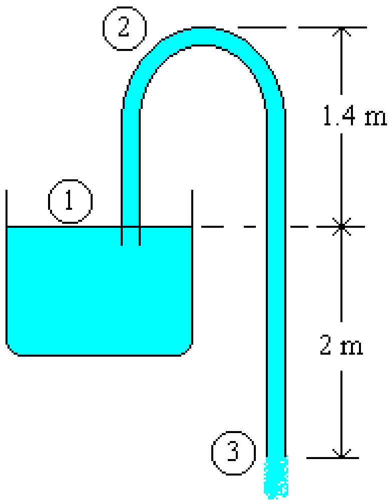
Hence

Fx = 25429 N

2.8 A siphon has a uniform circular bore of 75 mm diameter and consists of a bent pipe with its crest 1.8 m above water level and a discharge to the atmosphere at a level 3.6 m below water level. Find the velocity of flow, the discharge and the absolute pressure at crest level if the atmospheric pressure is 98.1 kN/m². Neglect losses due to friction.

Ans (0.0371 m³/s, 45.1 kN/m²)

Solution:



$$\frac{p_1}{\rho g}+\frac{V_1^{\ 2}}{2g}+z_1=\frac{p_3}{\rho g}+\frac{V_3^{\ 2}}{2g}+z_3$$

$$0+0+3.6=0+\frac{V_3^{\ 2}}{2g}+0$$

$$hence \rightarrow V_3=\sqrt{2x9.81x3.6}=8.404\ m/s$$

$$Q=AxV=\frac{\pi}{4}D^2xV=\frac{\pi}{4}x0.075^2x8.404=0.037\ m^3/s$$

$$\frac{98.1x10^3}{1000x9.81}+0=\frac{p_2}{\rho g}+5.4$$

$$\therefore P_2=98.1x10^3-5.4x9810=45126\ Pa$$

3 Chapter Three Tutorial Problems

3.1 If the vertical component of the landing velocity of a parachute is equal to that acquired during a free fall of 2m, find the diameter of the open parachute (hollow hemisphere) if the total weight of parachute and the person is 950N. Assume for air at ambient conditions, Density = 1.2 kg/m^3 and Cd = 1.35

Ans (6.169m)

Solution:

$$V_2^2 = V_1^2 + 2as$$
$$V = \sqrt{2x9.81x2} = 6.264\ m/s$$
$$NSL:$$
$$\sum F = 0$$
$$m.g - \frac{1}{2}\rho.V^2.A_f x C_d = 0$$

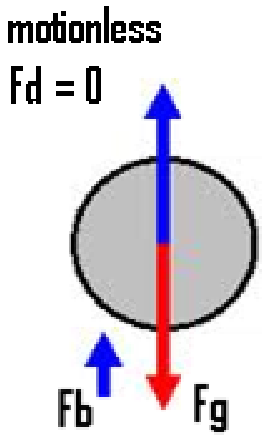
$$950 - \frac{1}{2}x1.2x6.264^2.(\frac{\pi}{4})D^2 x1.4 = 0$$

$$solve \rightarrow D = 6.17\ m$$

3.2 A buoy is attached to a weight resting on the seabed; the buoy is spherical with radius of 0.2m and the density of sea water is 1020 kg/m^3. Determine the minimum weight required to keep the buoy afloat just above the water surface. Assume the buoy and the chain has a combined weight of 1.2 kg.

Ans (33 kg)

Solution:



Since the system is stable and motionless, Newton's second law of motion reduces to:

Weight = Upthrust (Buoyancy)

Fb = Fg

Volume of sphere = (4.pi/3) x R^3 = 0.0335 m^3

The Upthrust is F_B = ρ_{fluid} x V_L x g =1020 x 0.0335x9.81= 335.3 N

Weight = 9.81 x(1.2 + Load)

Hence the payload = (335.3 / 9.81) – 1.2 = 32.98 kg

3.3 An aeroplane weighing 65 kN, has a wing area of 27.5 m^2 and a drag coefficient (based on wing area) C_D=0.02+0.061 xC_L^2. Assume for air at ambient conditions, Density = 0.96 kg/m^3. Determine the following when the craft is cruising at 700 km/h:

1. the lift coefficient
2. the drag coefficient, and
3. the power to propel the craft.

Ans (0.13, 0.021, 2040 kW)

Solution:

$$C_L = \frac{F_L}{(1/2)\rho.A.V^2} = \frac{65x10^3}{(1/2)x0.96x27.5xV^2}$$

$$C_D = 0.02 + 0.061xC_L^{\ 2} = 0.02 + 0.061x\left[\frac{65x10^3}{(1/2)x0.96x27.5xV^2}\right]^2$$

hence

$$F_D = \frac{1}{2}\rho.V^2.A_f xC_D = \frac{1}{2}x0.96xV^2x27.5x(0.02 + 14.79x10^5 xV^{-4})$$

Power

$$P = F_D x V = 0.264V^3 + 1.952x10^7 V^{-1}$$

$$V = 700\ km/h = \frac{700x1000}{3600} = 194.444\ m/s$$

hence

$$C_L = 14.79x10^5 x(194.444)^{-4} = 0.13$$

$$C_D = 0.02 + 0.061x0.13^2 = 0.021$$

$$P = 0.264x194.444^3 + 1.952x10^7 / 194.444 = 2041\ kW$$

3.4 A racing car shown below is fitted with an inverted NACA2415 aerofoil with lift to drag given as: $Cd = 0.01 + 0.008 \text{ x } Cl^2$

The aerofoil surface area is 1 m^2 and the car weight is 1 kN; the car maintains a constant speed of 40 m/s, determine at this speed:

1. The aerodynamic drag force on the aerofoil
2. The power required to overcome this drag force

Assume for air at ambient conditions, take Density = 1.2 kg/m^3

Ans (18 N, 0.7 kW)

Solution:

$$C_L = \frac{F_L}{(1/2)\rho.A.V^2} = \frac{1x10^3}{(1/2)x1.2x1x40^2} = 1.0417$$
$$C_D = 0.01 + 0.008xC_L{}^2 = 0.01 + 0.008x1.0417^2 = 0.01868$$

$$F_D = \frac{1}{2}\rho.V^2.A_f xC_d = \frac{1}{2}x1.2x1x40^2 x0.01868 = 17.9\ N$$

$$Power = F_D xV = 17.9x40 = 0.717\ kW$$

3.5 Air flows over a sharp edged flat plate, 3m long and 3m wide at a velocity of 2 m/s.

1. Determine the drag force
2. Determine drag force if the plate was mounted perpendicular to the flow direction assume Cd = 1.4.

For air, take density as 1.23 kg/m^3, and kinematic viscosity as 1.46x10^{-5} m/s^2.

Ans (0.05N, 31N)

Solution:

1.

$$\mathrm{Re}_l = \frac{V_\infty x L}{\upsilon} = \frac{2x3}{1.46x10^{-5}} = 4.11x10^5 < 5x10^5 \rightarrow Laminar$$

$$C_D = \frac{1.328}{(\mathrm{Re})^{0.5}} = 0.00207$$

$$F_D = \frac{1}{2}\rho.A.V_\infty{}^2 x C_D = \frac{1}{2}x1.23x(3x3)x2^2\,x0.00207 = 0.045\ N$$

2.

$$Cd = 1.4$$

$$F_D = \frac{1}{2}\rho.A.V_\infty{}^2 x C_D = \frac{1}{2}x1.23x(3x3)x2^2\,x1.4 = 30.4\ N$$

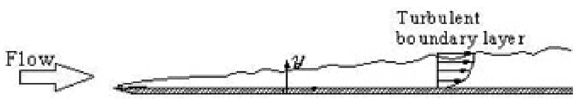
3.6 (a) An airplane wing has a 7.62m span and 2.13m chord. Estimate the drag on the wing (two sides) treating it as a flat plate and the flight speed of 89.4 m/s to be turbulent from the leading edge onward.

(b) Determine the reduction in power that can be saved if the boundary layer control device is installed on the wing to ensure laminar flow over the entire wing's surface.

For air, take density as 1.01 kg/m^3, and kinematic viscosity as 1.3x10^{-5} m/s^2.

Ans (358 N, 86%)

Solution:



$$C_D = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \quad \text{laminar,} \quad \mathrm{Re}_L < 5\times10^5$$

$$C_D = \frac{0.074}{[\mathrm{Re}_L]^{\frac{1}{5}}} \quad \text{turbulent,} \quad 5\times10^5 \le \mathrm{Re}_L < 10^7$$

a)

$$Re_l = \frac{V_\infty x L}{\upsilon} = \frac{89.4x2.13}{1.3x10^{-5}} = 1.265x10^7 > 5x10^5 \rightarrow Turbulent$$

$$C_D = \frac{0.074}{(Re)^{0.2}} = 0.00273$$

$$F_D = \frac{1}{2}\rho.A.V_\infty{}^2 xC_D = \frac{1}{2}x1.01x(2x7.62x2.13)x89.4^2 x0.00273 = 357.6\ N$$

$$Power = F_D xV = 357.6x89.4 = 31.969\ kW$$

b) *laminar.*

$$C_D = \frac{1.328}{(1.265x10^7)^{0.5}} = 0.00037$$

$$F_D = \frac{1}{2}\rho.A.V_\infty{}^2 xC_D = \frac{1}{2}x1.01x(2x7.62x2.13)x89.4^2 x0.00037 = 48.9\ N$$

$$Power = F_D xV = 48.9x89.4 = 4.372\ kW$$

$$\%reduction = 86.3\%$$

4 Chapter Four Tutorial Problems

4.1 Assuming the ideal gas model holds, determine the velocity of sound in

a) air (*mwt* 28.96) at 25°C, with $\gamma = 1.4$,
b) argon (*mwt* 39.95) at 25°C, with $\gamma = 1.667$.

Ans: [346 m/s, 321.5 m/s]

Solution:

For air at 25 °C

$$R = 8314.4/ 28.96 = 287.096 \text{ J/kgK}$$

$$a= [\gamma R T]^{1/2} = (1.4 (287.096 \text{ J/K kg}) (298\text{K}))^{1/2} \quad = 346.029 \text{ (m/s)}$$

For Argon

$$R = 8314.3 / 39.95 = 208.117 \text{ J/kgK}$$

$$a= [\gamma R T]^{1/2} = (1.667 (208.117 \text{ J/K kg}) (298\text{K}))^{1/2} \quad = 321.536 \text{ (m/s)}$$

4.2 An airplane can fly at a speed of 800km/h at sea-level where the temperature is 15°C. If the airplane flies at the same Mach number at an altitude where the temperature is -44°C, find the speed at which the airplane is flying at this altitude.

Ans: [195m/s]

Solution:

$a= [\gamma R T]^{1/2}$

$$= (1.4 \text{ x}287\text{x}(273+15))^{1/2}$$
$$= 346.17 \text{ (m/s)}$$

V_1= 800 km/h = 800 x 1000/3600 = 222.222 m/s
M = V/a = 222.222 / 346.17 = 0.642

$a_2 = [\gamma R T]^{1/2}$

$$= (1.4 \text{ x}287\text{x}(273-44))^{1/2}$$
$$= 303.33 \text{ (m/s)}$$

Hence

$$V_2 = M \text{ x } a_2 = 303.33 \text{ x } 0.642 = 195 \text{ m/s}$$

4.3 A low flying missile develops a nose temperature of 2500K when the ambient temperature and pressure are 250K and 0.01 bar respectively. Determine the missile velocity and its stagnation pressure. Assume for air: γ = 1.4. Cp = 1005 J/kgK

Ans: [2126 m/s, 31.6 bar]

Solution:

$$Cp = Cv + R$$

$$Cp = Cp/\gamma + R$$

hence

$$R = \frac{(\gamma - 1)}{\gamma} xCp = \frac{1.4 - 1}{1.4} x1005 = 287.143\ J/kgK$$

$$T_o - T = V^2/2Cp$$

$$V = \sqrt{2xCpx(T_o - T)} = \sqrt{2x1005x(2500 - 250)} = 2126.67\ m/s$$

$$\frac{P_o}{P} = [1 + \frac{V^2}{2.Cp.T}]^{\frac{\gamma}{\gamma-1}}$$

$$P_o = Px\left[1 + \frac{V^2}{2Cp.T}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= 0.01x\left[1 + \frac{2126.67^2}{2x1005x(273 + 250)}\right]^{\frac{1.4}{0.4}} = 31.6\ bar$$

4.4 An airplane is flying at a relative speed of 200 m/s when the ambient air condition is 1.013 bar, 288 K. Determine the temperature, pressure and density at the nose of the airplane. Assume for air: γ = 1.4, density at ambient condition = 1.2 kg/m^3 and Cp =1005 J/kgK.

Ans: [T_o=307.9K, P_o=1.28 bar, ρ =1.42 kg/m^3]

Solution:

$$T_o = T + \frac{V^2}{2xCp} = 288 + \frac{200^2}{2x1005} = 307.9\ K$$

$$\frac{P_o}{P} = [1 + \frac{V^2}{2.Cp.T}]^{\frac{\gamma}{\gamma-1}}$$

$$P_o = Px\left[1 + \frac{V^2}{2Cp.T}\right]^{\frac{\gamma}{\gamma-1}} = 1.013x\left[1 + \frac{200^2}{2x1005x288}\right]^{\frac{1.4}{0.4}} = 1.28\ bar$$

$$\rho_o = \frac{P_o}{RxT_o} = \frac{1.28x10^5}{287x307.9} = 1.448\ kg/m^3$$

$$\rho_o = \rho\ x[\frac{P_o}{P}]^{\frac{1}{\gamma}} = 1.2x\left[\frac{1.28}{1.013}\right]^{1/1.4} = 1.418\ kg/m^3$$

5 Chapter Five Tutorial Problems

5.1 A small-scale hydraulic power system has an elevation difference between the reservoir water surface and the pond water surface downstream of the turbine is 10 m. The flow rate through the turbine is 1 m^3/s. The turbine/generator efficiency is 83%. Determine the power produced if:

a) Flow losses are neglected.
b) Assume friction loss equivalent to 1 m head.

Ans: (81 kW, 73 kW)

Solution:

a)

$$losses = 0m$$

$$P_A = \rho\, g\, Q\, H\, \eta = 1000x9.81x1x10x0.83 = 81.4\ kW$$

b)

$$losses = 1m$$

$$P_B = 1000x9.81x1x(10-1)x0.83 = 73.26\ MW$$

5.2 A hydro-electric power plant based on the Loch Sloy in Scotland has an effective head of 250 metres. If the flow rate of 16 m^3/s can be maintained, determine:

a) the total power input to the turbine assuming a hydraulic efficiency of 98%; and
b) the pressure difference across the turbine.

Ans: (38 MW, 2.4 MPa)

Solution:

The power available from water is

P = ρ Q g h x η
= 1000 x 16 x9.81 x 250 x0.98
= 38455.2 kW which is 38 MW.

The power can also be expressed as = P = Q x Δp

Hence

$$\Delta p = P / Q = 38455200 / 16 = 2.403 \text{ MPa}$$

5.3 A proposed hydropower plant to be built using a reservoir with a typical head of 18m and estimated power of 15 MW. You are given the task to select an appropriate type of turbine for this site if the generator requires the turbine to run at a fixed speed of 120 rpm.

Ans: (Ns=396, Francis or Kaplan)

Solution

Type of Turbine	Specific speed range $Ns = N \dfrac{P^{1/2}}{(h)^{5/4}}$
Francis	70–500
Propeller	600–900
Kaplan	350–1000
Cross-flow	20–90
Turgo	20–80
Pelton, 1-jet	10–35
Pelton, 2-jet	10–45

$$Ns = N \frac{P^{1/2}}{(h)^{5/4}} = 120x\frac{15000^{0.5}}{18^{1.25}} = 396.4$$

hence – Francis or Kaplan

Solution of Mock Exam

Sample Examination Paper

STAFFORDSHIRE UNIVERSITY

Class Test – Fluid Mechanics

Module Tutor T. Al-Shemmeri

This Paper contains TEN questions. Attempt all questions.
A formulae sheet is provided.
Place your Answers in the space provided. No detailed solution required.
Print your name on every page. Submit all together for marking.

MARKING GRID LEAVE BLANK PLEASE

question	1	2	3	4	5	6	7	8	9	10	total
1st Marker											
2nd marker											

Agreed percentage	
Recommended grade	

Question One

List TWO types of instrument used to measure the pressure of a toxic fluid contained in a sealed tank. Complete the table below:

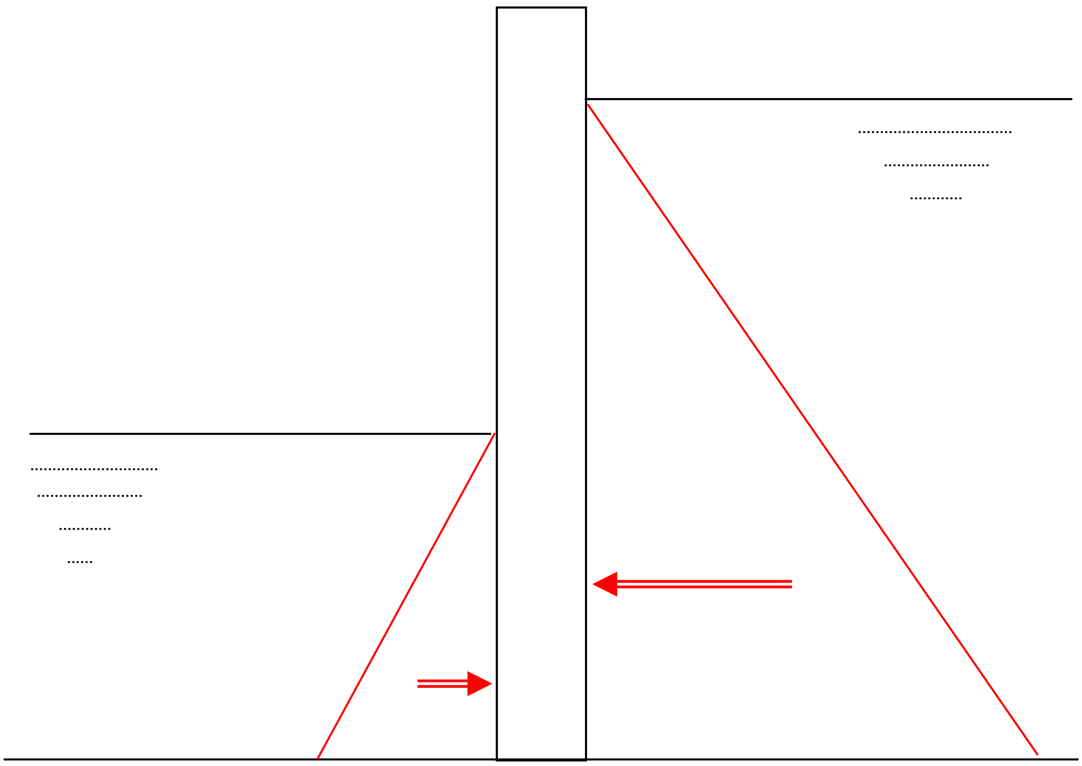
Type	Principle	Marks
Standard Bourdon gauge	Expansion of a helical element when stressed by the action of pressure/force	/ 4 marks
Piezometer – closed	A **piezometer** is a device used to measure static liquid pressure in a system by measuring the height to which a column of the liquid rises against gravity	/ 4 marks

Total (8 marks)

Question Two

a) Draw (not to scale) the pressure distribution of the water on the dam shown below:

(6 marks)



b) Indicate on the sketch, the direction of the resultant force on the dam?

(2 marks)

c) Approximately, indicate the position of the centre of pressure on both sides.

(2 marks)

Total (10 marks)

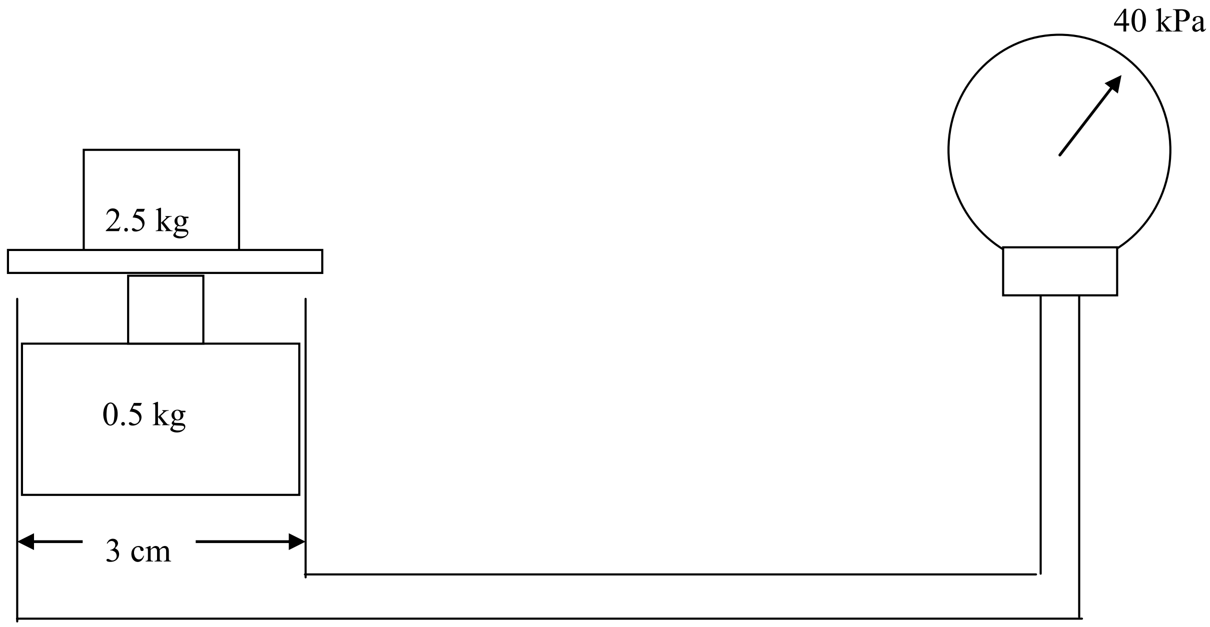
Question Three

List Three methods used to improve the resolution of detecting a small pressure reading in a manometer. Complete the table below:

Method	Principle	Marks
Incline one limb	Trigonometry will mean the sensitivity can be improved by reading the longest side of a triangle. Read (Lx Sin(theta)) instead of reading (h)	/ 3 marks
Use different size (cross-sectional area) for either side of the manometer.	Help is derived from the continuity law, or displaced volume on either side to remain the same, A. z = constant, so z2 can be much larger if A2 is smaller than A1.	/ 3 marks
Use different fluids with appreciably different density values.	This uses the conservation of mass, for the displaced column on either side of the manometer.	/ 3 marks

Total (9 marks)

Question Four



Complete the table below:

Theoretical reading of the pressure $P = F / A = (2.5+0.5)x9.81 /(3.14x0.03^2/4)$	41635	/ 4 marks
% error =[(41.635-40.000)/41.635]x100%	3.93%	/ 3 marks
The maximum load if the gauge limit is 100 kPa $F = P \times A = 100x10^3 x(3.14x0.03^2/4)$	70.686 N = 7.2 kg	/ 3 marks

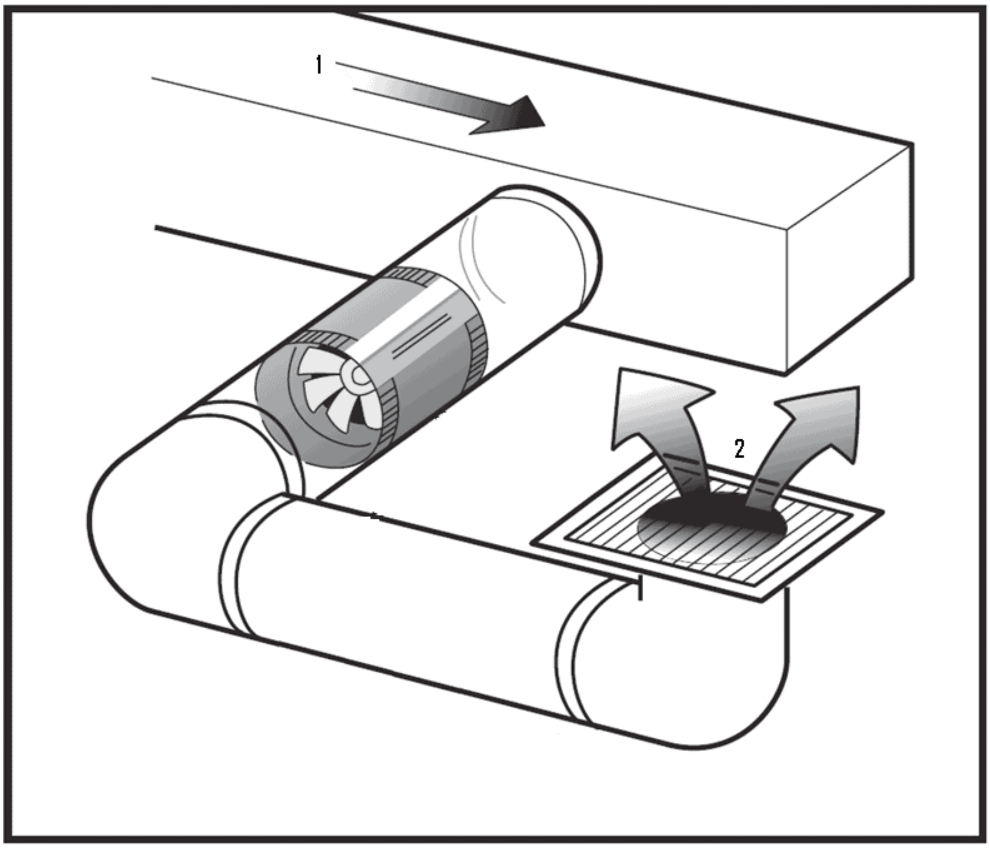
Total (10 marks)

Question Five

If the fan, below, circulates air at the rate of 0.30 m^3/s, determine the velocity in each section. Complete the table below.

Section	Dimensions m	Area m^2	Velocity m/s		Marks
1	0.25 square	$0.25 x 0.25 = 0.0625\ m^2$	V1	=Q/A = 0.3/0.0625 = 4.8 m/s	/ 5 marks
2	0.20 diameter	$(3.14/4) x 0.2^2 = 0.0314\ m^2$	V2	=Q/A = 0.3/0.0314 = 9.5 m/s	/ 5 marks

Total (10 marks)



Question Six

Oil of relative density 0.90 flows at the rate of 100 kg/s in a horizontal pipe of 200 mm diameter, 1 km long. If the friction factor for the pipe is 0.006, complete the following table:

Quantity	Value	Units	Marks
flow velocity m = density x A x V hence V = m / (den x A)	3.537	m/s	/ 3 marks
frictional head loss h = (4f L/D) x V^2/2g	= (4x0.006x1000/0.2)x (3.537^2/19.62 = 76.5	m	/ 3 marks
frictional pressure loss P = den x g x h	= 900x9.81x76.5 = 675	kPa	/ 2 marks
energy to overcome friction E = m.g.h	= 100x9.81x76.5 = 75	kW	/ 2 marks

Total (10 marks)

Question Seven

Show that Bernoulli's equation is dimensionally homogeneous

3 marks for the p-term,
3 marks for the v-term, and
2 marks for the z-term and
2 for stating that all dimensions have/have not the same dimensions

Total (10 marks)

Solution:

Take each term separately

(a) $\frac{p}{\rho g} \Leftrightarrow \frac{MLT^{-2}}{(ML^{-3})(LT^{-2})} \Leftrightarrow L \Leftrightarrow m$

(b) $\frac{V^2}{2g} \Leftrightarrow \frac{(L/T)^2}{(0)(LT^{-2})} \Leftrightarrow L \Leftrightarrow m$

(c) $Z \Leftrightarrow (L) \Leftrightarrow m$

All terms have the same units of meters. The equation is therefore dimensionally balanced.

Question Eight

Oil (relative density 0.85, kinematic viscosity 80cs) flows at the rate of 90 tonne per hour along a 100 mm bore smooth pipe. Determine for the flow:

Quantity	Value	Marks
flow velocity A= (pi/4)xD^2 m = 90 t/h = 90 x 1000/3600 V = m / A/ density	 0.00785 m^2 25 kg/s 3.745 m/s	/ 5 marks
Nature of the flow New = 80 cs = 80×10^{-6} m^2/s Re = den x V x D/ mu = V x D / New	 Re =3.745 x 0.1 / 80×10^{-6} Re = 4682 Hence flow turbulent	/ 5 marks
frictional factor f = 0.079 / $Re^{0.25}$	 f = 0.0095	/ 2 marks

Total (12 marks)

Question Nine

List two instruments for measuring the flow rate of air through a rectangular duct.

Method	Principle	marks
Hot wire Anemometer	use a very fine wire (on the order of several micrometres) electrically heated up to some temperature above the ambient. Air flowing past the wire has a cooling effect on the wire. Uses the relationship between the electrical resistance of the wire and the flow speed.	/ 4 marks
Pitot-static tube	Relies on the measured static and dynamic pressures to determine the fluid velocity using Bernoulli's equation.	/ 4 marks

Total (8 marks)

QUESTION TEN

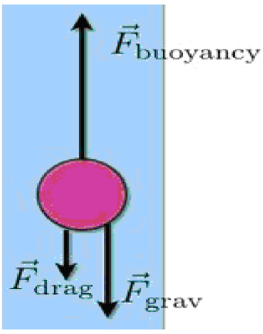
Draw the body force diagram for a parachute jumper.

If the vertical component of the landing velocity of a parachute is 6 m/s, find the total weight of the parachutist and the parachute (hollow hemisphere Diameter 5m).

Assume for air at ambient conditions, Density = 1.2 kg/m^3 and Cd = 2.3

For correct body force diagram	**/2 marks**
For correct use of formula	**/ 6 marks**
For correct answer	/ 2 marks
	Total (10 marks)

Solution



The free body diagram shows that the body is affected by 3 forces.

Newton's second law of motion states that when the object is moving at steady state then the net force on the object must be zero

Hence F g = Fb + Fd

Since the parachute is open, the buoyancy force will be negligible and only due to the body of the person, which is small in comparison to the parachute size. Fb = 0

$$NSL:$$
$$\sum F = 0 \rightarrow F_w - Fd = 0$$
$$F_w - \frac{1}{2}\rho.V^2.A_f xC_d = 0$$
$$F_w = \frac{1}{2}x1.2x6^2.(\frac{\pi}{4})x5^2x2.3 = 975\ N = 99.4\ kg$$

Formulae Sheet

FLUID STATICS:

$$P = \rho g h$$

CONTINUITY EQUATION:

mass flow rate $m = \rho A V$

volume flow rate $Q = A V$

Energy Equation

$$(P/\rho g) + (V^2/2g) + Z = \text{constant}$$

DARCY'S EQUATION

$$H_f = (4 f L/D)(V^2/2g)$$

FRICTION FACTOR FOR A SMOOTH PIPE

$f = 16/ Re$ if $Re < 2000$

$f = 0.079 / Re^{0.25}$ if $Re > 4000$

MOMENTUM EQUATION

$F = m (V_2 \cos\theta - V_1)$

Drag FORCE = $Cd \times (1/2) \times \rho . A.V^2$

FLUID POWER

$E = \rho g h Q$